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## **Modeling criteria used for computational model of transformer**

**Abstract** – *The criteria of physical modeling the electromagnetic fields in transformers, taking into account Maxwell and consecutive equations, were presented. The full or different particular approximation models have been formulated by respective modeling scales. The application of modeling scales in computational model were discussed*

**Keywords:** modeling criteria of electromagnetic fields, computational models of transformer

### **Introduction**

Despite the development of computer techniques, the possibility of solution the electric and magnetic fields in all regions of transformer with high accuracy of modeling will be yet distant.

Actually, the different approximation of computational details, properties of media and secondary reaction on excitation values are assumed. The most popular methods: Reluctance Network Method (RNM), Finite Difference Method (FDM), Finite Element Method (FEM) were developed for 3D calculations. Especially FEM method is used in commercial packages becoming very popular in engineering applications (ANSYS, OPERA, FLUX etc) [2,3].

It is rather easy to adapt professional 3D MES packages for electrical machines calculation because their rotational or axial symmetry. It is not the same case for transformer design. Therefore the advance in field calculation of transformers in the scale of linear dimensions  $m = 1$  was in the past not satisfied and it seems in modern times in quite similarly. In mathematical modeling, the theoretical possibility exists to reach the virtual solutions in a sense of so called "Full physical model" by same changing the parameters, even their values was physically unrealistic. This simulation procedure is under investigation.

During last years, it was observed the important advance of computer aided preparation of design documentation. The AUTOCAD or AUTODESK systems generate virtual forms of devices with an accurate geometric representation of some details of transformer construction. However, a common input into the programs both: the boundary condition on the media interface, with various degrees of discretization and the regard of eddy currents in core laminations as well as in structural parts present yet some barrier to pass.

The growth of computer computation capacity was the reason of the weakness of the interest in physical modelling. From another side, the experience collected in many years in transformer factories has had the influence on this matter. The observed globalization of production by international consortia enables to increase constructional data basis and unify calculating methods.

In this work, author proposes the use of criteria elaborated for physical modeling of electromagnetic fields in transformers, taking into account different models (worked in the past years) for computational models in recent computation possibilities, realized by

commercial packages. The geometrical scale of dimensions  $m_l$  is proposed. The calculating packages contain limiting number of discretization points of analyzed 3D area. The degree of absolute discretization decreases in objects with high geometrical size. The representation of real device by the model with geometrical scale  $m_l < 1$  provokes the increase the discretization degree of calculating area.

### Physical models description

Theory of physical modeling gives an important contribution to design and development of transformer technology. For beginning of XX century the so called growth law of transformer as an particular case of physical modeling was applied to put forward numerous commercial offers on transformer markets. In many laboratories of electrical engineering, it is possible to find the transformers which may be qualified as the growth law models [6]. In the sixties years of XX century, particular in former Soviet Union, the theory of physical modeling of electromagnetic fields was developed. The modeling, taking into account magnetic nonlinearity was worked by the author in Institute of Electrical Machines and Transformers (now Institute of Mechatronics and Information Systems), Lodz University of Technology [4,5,6,7]. The short presentation of the different models which may be applied in computational models of transformer systems will be present as follows.

### Full model

From Maxwell equations

$$(1) \quad \text{rot}H = \sigma E + \frac{\partial D}{\partial t} + Ji$$

$$(2) \quad \text{rot}E = -\frac{\partial B}{\partial t}$$

and constitutive equations

$$(3) \quad B = \mu H$$

$$(4) \quad D = \varepsilon E$$

The following criteria equations may be deduced

$$(5) \quad m_\varepsilon m_\mu m_f^2 m_l^2 = 1$$

$$(6) \quad m_\sigma m_\mu m_f m_l^2 = 1$$

$$(7) \quad \frac{m_\mu m_l}{m_H} = 1$$

The execute simultaneously equations (5) and (6), the criterion (8) must be assumed.

$$(8) \quad m_\varepsilon m_f = m_\sigma$$

Assuming the same insulation materials  $m_\varepsilon = 1$ , we obtain

$$(9) \quad m_f = m_\sigma$$

Assuming the same level of magnetic saturation in ferromagnetic area (in the model and original) we get  $m_H = m_B = m_\mu = 1$ . It is possible to write

From (7)

$$(10) \quad m_{ji} = \frac{1}{m_l}$$

From (6)

$$(11) \quad m_f = \frac{1}{m_l}$$

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From (8)

$$(12) \quad m_{\sigma} = \frac{1}{m_l}$$

The application in both the model and the original, the same dielectric, magnetic and conducting materials, corresponding to the criterion formulation as above, is possible in computational model without difficulties.

### **Dielectric model**

Hitherto, physical modeling was made with approximate assumptions. The models which expose dielectric phenomena were built with execution criterion (5), only.

Consequently  $m_f = \frac{1}{m_l}$ . This models (known unjustly as the electromagnetic physical

models) were prepared in the praxis for investigation the electric potential distribution in steady state or shock-wave distribution along transformer windings.

### **Eddy current model**

Investigations of additional losses in structural elements evoked by leakage flux were performed on so called "Eddy current models" under assumption criterion (6) as

well as  $m_{\sigma} = m_{\mu} = 1$ ; it is  $m_f = \frac{1}{m_l^2}$ , in consequence. It needs to attend that scale of

magnetic strength  $m_H = m_l$  that does not permit to take into account the same level of magnetic saturation in the model as in the original. In the work [5], the author has proposed the maintenance  $m_H = 1$ , by the way of intensification of cooling of transformer model windings that has led to avoid difficulties related with recalculation of power losses from the model to original without problem of magnetic nonlinearity.

### **Model deduced from growth low of transformer**

The criterion (6) is executed in physical modeling only for  $m_l = 1$  because the assumed scale in the model are equal to  $m_{ji} = m_f = m_{\sigma} = m_{\mu} = 1$ . The approximation deals with the different scale in the magnetic core  $m_H = 1$  and without core  $m_H = m_l$ .

The results of calculation the different scales of physical quantities modelling according with criteria described in [1 – 4] are presented in Table 1. Table 2 collect the scale modelling of main transformer parameters which may be used for recalculation computer results for original values.

In ferromagnetic parts of the transformer with nonlinear dependence  $B(H)$  and  $m_H$  not equal to 1, we obtain also nonlinear functions  $m_{\mu}(m_H)$  or  $m_B(m_H)$  as additional complication of recalculation procedure for eligible physical quantities or transformer parameters. This is an important problem as well in magnetic core as in structural parts of transformer. For this reason, the full and eddy current models are preferable for testing in the future computer calculations.

The main assumption in this work, the scale of winding number turns  $m_N = 1$  is justified by design documentation of transformer without the construction change possibility of responsible insulation systems.

The calculation methods of power losses in a magnetic core, using traditional analytic formulae [1] adequate to measured losses arising in real transformers are not satisfied. They are some reasons depending on different compounds of laminations, manufacture technology, sheet cutting and core composition. The theoretical troubles in the field of core loss generation are observed this time also.

In spite of numerous experiences, the universal calculating method which may be applied with good accuracy in the praxis not yet exists. From author's investigation may be deduce, that assuming the elliptic hysteresis loop as an equivalent of static characteristic of lamination material, may be an interesting way to generalization of this problem [8]. The calculated power losses, taking into account simultaneously the hysteresis and eddy currents obtained for defined frequency (for instance 50 Hz) should be recalculated for another frequency after introduction the coefficient  $A_n$  depending on the average magnetic flux density  $B_{av}$  in a lamination and the angle between magnetization axis and main rolling direction of magnetic steel. The means of  $A_n$  fixing was described, into others, in the work [9].

Table 1. Scale modeling of physical quantities with assumption  $m_N = 1$  (scale of winding number turns).

No	Appellation	Signature	Models				Remarks
			Full	Dielectric	Eddy Current	Growth low	
1	Linear dimensions	$m_l$	$m_l$	$m_l$	$m_l$	$m_l$	
2	Dielectric permeability	$m_\epsilon$	1	1	1	1	
3	Frequency	$m_f$	$\frac{1}{m_l}$	$\frac{1}{m_l}$	$\frac{1}{m_l^2}$	1	
4	Electric conductivity	$m_\sigma$	$\frac{1}{m_l}$	1	1	1	
5	Density of excitation currents	$m_{ji}$	$\frac{1}{m_l}$	1	$\frac{1}{m_l}$	1	
6	Magnetic field strength	$m_H$	1	$m_l$	1	1	in magnetic core
						$m_l$	outside core
7	Electric field strength	$m_E$	1	$m_\mu(m_H)m_l$	$\frac{1}{m_l}$	$m_\mu(m_H)m_l^2$	induced in structural parts
			1	$m_l$	$m_l$	1	in dielectrics
			1	1	$\frac{1}{m_l}$	1	along current lines in winding
8	Magnetomotive force	$m_\theta$	$m_l$	$m_l^2$	$m_l$	$m_l$	in magnetic core
						$m_l^2$	outside core for $m_\mu = 1$
9	Magnetic induction	$m_B$	1	$m_B(m_H)**$	1	1	in magnetic core and dielectrics
						$m_B(m_H)$	structural parts
10	Magnetic permeability	$m_\mu$	1	$m_\mu(m_H)*$	1	1	in magnetic core and dielectrics
						$m_\mu(m_H)$	structural parts
11	Magnetic flux	$m_\Phi$	$m_l^2$	$m_\mu(m_H)m_l^3$	$m_l^2$	$m_l^2$	in magnetic core
						$m_l^3$	outside core for $m_\mu = 1$

\* - nonlinear function  $m_\mu(m_H) = f_1(m_H)$

\*\* - nonlinear function  $m_B(m_H) = f_2(m_H)$

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In Table 2 were presented the scale modeling of core losses deduced from author's investigation with assumption the same thickness of laminations in a model and original  $m_d = 1$ . The values used in formulae were described in the papers [7,8,9].

Table 2. Scale modeling of transformer parameters with assumption  $m_N = 1$   
(scale of winding number turns).

No	Appellation	Signature	Models			
			Full	Dielectric	Eddy Current	Growth low
1	Excitation current in windings	$m_I$	$m_I$	$m_I^2$	$m_I$	$m_I^2$
2	Induced voltage in winding	$m_U$	$m_I$	$m_\mu(m_H)m_I^2$	1	$m_I^2$
3	Apparent power	$m_S$	$m_I^2$	$m_\mu(m_H)m_I^4$	$m_I$	$m_I^4$
4	Resistance	$m_R$	1	$\frac{1}{m_I}$	$\frac{1}{m_I}$	$\frac{1}{m_I}$
5	Inductance	$m_L$	$m_I$	$m_I$	$m_I$	$m_I$
6	Reactance	$m_X$	1	1	$\frac{1}{m_I}$	$m_I$
7	$tg \varphi_z = \frac{X}{R}$	$m_{tg\varphi_z}$	1	$m_I$	1	$m_I^2$
8	Capacitance	$m_C$	$m_I$	$m_I$	$m_I$	$m_I$
9	Surface density of structural power losses	$m_P$	1	$\sqrt{m_\mu(m_H)}m_I^3$	$\frac{1}{m_I}$	$\sqrt{m_\mu(m_H)}m_I^2$
10	Winding power losses	$m_{Pw}$	$m_I^2$	$m_I^3$	$m_I$	$m_I^3$
11	Power losses in magnetic core	$m_{Pc}$	$m_I^3 \frac{\xi_H\left(\frac{kd}{m_I}\right)}{\xi_H(kd)}$	$\frac{m_I^3}{\sqrt{m_I}} \frac{\xi_H\left(\frac{kd}{m_I}\right)}{\xi_H(kd)} m_{A_N}$	$m_I^2 \frac{\xi_H\left(\frac{kd}{m_I}\right)}{\xi_H(kd)}$	$m_I^3$
12	Power losses in structural parts	$m_{Sp}$	$m_I^2$	$\sqrt{m_\mu(m_H)}m_I^{\frac{7}{2}}$	$m_I$	$\sqrt{m_\mu(m_H)}m_I^4$

## Conclusions

The trends for application of commercial packages for full design of transformers, taking into account the electromagnetic, dielectric, thermal, acoustic etc. problems are observed for many years. Despite enlargement of calculating possibilities by introduction coupled fields solvers, the perspectives of total computation design of electromagnetic devices will be limited because numerous factors defined by structural solutions and constructional material conditions. In a sense of computation techniques, the mathematical and geometrical model approximation from point of view of good accuracy of calculations will be always current and important problem for designers.

In this work the different models of electromagnetic fields in transformers, developed on the base of physical models analysis were presented. The "Full and Eddy current model" criteria, described in the paper, may be used in computational models of transformer prepared in the scale of linear dimensions  $m_l$ . The known limitation of finite elements in the volume of calculating area in recent commercial packages may be an encouragement for computation the electromagnetic field in models with assumption the scale of linear dimensions  $m_l < 1$ . In this way, it is possible to obtain the increase of the level of absolute discretization of the calculation area. Independent on this idea, the modeling scales presented in the paper are rightful as well for models defined by  $m_l < 1$  as by  $m_l > 1$ .

In conclusion, it is necessary to express the hope for the further advance in the field of electromagnetic computation of transformers.

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